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CAVITATING PENETRATION OF A PLASTIC MEDIUM BY BODIES OF MINIMUM RESISTANCE†

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An approximate approach to the description of the deep penetration of a plastic medium by elongated bodies [1] is developed. The results obtained in [2–5] are generalized to another geometry and to a wider range of velocities, which is subdivided into intervals. An attempt is made to provide an averaged description of the motion in each of the intervals assuming quasistationarity, and the formula for the depth of penetration is analysed. Lavrent'yev's idea on the closeness of the velocity and stress fields during the high-velocity motions of bodies in a solid medium and in a liquid, which has been translated into the language of asymptotic representations [5], has been further developed in an hypothesis on the closeness of the configurations of bodies of minimum resistance in a plastic medium and a liquid subject to the condition of the smallness of the ratio of the strength (resistance) force to the hydrodynamic drag force. The advantage of tubular bodies compared with solid bodies with respect to mass and depth of penetration is indicated. The bounds of the limiting penetration depths are estimated.

1. AN ABSOLUTELY rigid, elongated and bulky or thin-walled hollow body of revolution (a projectile or penetrator) of length L penetrates into a homogeneous, isotropic, viscoelastic half-space along a normal to its surface. We will determine the dependence of the depth of penetration H_0 on the initial velocity of the body v_0 and the other parameters of the problem. The main differences between this investigation and previous theoretical investigations [4, 5] are the wider range of velocities, including supersonic velocities, and the allowance for the separation of the flow of the medium close to the nose part of the body. Moreover, the phenomenon of flow separation is made substantial use of when optimizing the penetration process with a choice of configuration close to the shape of the body of minimum resistance. Since a satisfactory solution of such a complex multiparametric and extremely non-linear problem is not possible using either analytic or numerical methods, we have used the well-known approach of the summation of asymptotic forms which has been indirectly proved theoretically [4, 5] and experimentally [3].

The medium is characterized by a density ρ_0, a velocity of propagation of an elastic wave c and a von Mises flow limit τ. Young's modulus E is used for the normalization instead of the supposedly more accurate modulus of compression from considerations of the convenience of the use of reference material and the estimatory nature of the results obtained. Bearing in mind the applications to terradynamics, with regard to the loose soil we will assume that packing takes place at a distance from the body and that ρ_0 is the density in the packed state. Additionally, the shape and mass distribution over the length of the penetrator are such that it moves in a straight line, is stable, does not rotate, its high mean density $\rho_1 \gg \rho_0$ and the velocity as well as its elongation (1.1) ensure deep penetration ($H_0 \gg L$). The body itself is assumed to be thin and

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blunt and, it is assumed, in fact, that the geometry of its surface, beyond a leading part of length l_0, is weakly profiled

$$R'(x),\ h'(x) << 1,\ l_0 < x < L;\ h << R,\ l_0 << L \tag{1.1}$$

where R and h are the external radius and the thickness of the walls.

We additionally make the following hypotheses.

1. The motion of a body in a medium can be considered as a quasisteady process. The elongation and relatively high density of the penetrator promote a reduction in the accelerations while, in the treatment of the experimental data in [3] the insignificance of the added mass for similar configurations was noted. We shall ignore this added mass below.

2. The initial segment of the entry (the effect of the free surface), as well as other transitional processes, are not taken into account.

3. We shall base our investigation upon the well-known hypothesis of the closeness of the velocity and stress fields to the analogous hydrodynamic fields.

It has been shown in [4, 5] that this closeness is asymptotic, and corrections, due to the strength, to the integral characteristics of the steady-state flow around blunt and sharpened smooth bodies by an elastoplastic fluid (transonic and subsonic, respectively) have been calculated. Moreover, the drag forces are described to a first approximation by a formula of the type of the Poncelet formula [2], the coefficients in which are non-trivially related to the constitutive parameters of the medium and the shape of the body. Starting from an asymptotic approach, it is natural to put forward the hypothesis that the configuration of the body of minimum drag in a solid medium will be close (in an asymptotic sense) to the profile of the same body in a liquid. It is well known from hydrodynamics that a body which is inserted into an expanding cavern behind a disc minimizes the drag [6]. A number of experiments indicate the closeness of the laws of expansion of caverns in liquids and in a solid (see [7], for example). The strength naturally reduces the rate of expansion of a cavern and, furthermore, forms a stagnation zone ahead of the blunt leading part of the penetrator which becomes thinner as the velocity increases. It is therefore necessary to correct the contour by passing from hydrodynamics to terradynamics with the aim of minimizing the drag and, first of all, account must be taken of the narrower cavern behind the body in a solid medium compared with a liquid.

Thus, we assume that the body has an optimal profile in the sense that separation of the flow of the medium from the surface of the body occurs in a section $x = x_*$ $(R = R_*,\ h = h_*)$ over a significant section of the trajectory of the body at values of the dimensionless velocity (the Mach number) $M_* < M < M_0$, $M = \upsilon/c$, where υ is the free flow velocity and M_* is the critical (reference) value of the Mach number at which a change in the conditions occurs. It is assumed that the flow does not come into contact with the surface over the length $x_* < x < L$ when $M > M_*$. When $M < M_*$, the penetrator moves under conditions of total contact. We emphasize that the aim of this paper is to describe the integral characteristics of bodies of minimum resistance in a terrestrial medium while pointing out certain general principles of the construction of optimal configurations. It is necessary to carry out the search for the actual profile, which depends on many factors, on the basis of a text experiment and a corrective calculation.

4. In the case of a tube, we adopt the hypothesis of an isolated element in the form of a narrow meridional strip, cut out of the tube, the length of the arc of which a is such that $h \ll a \ll R$. In other words, we implement the a priori assumption of the closeness (in the asymptotic sense of (1.1)) of the stress and velocity fields around the walls of the tube to the analogous fields during the flow around a planar body with a profile identical to that of the meridional section of the tube. The effect of the differences in these fields, which arise when the flow separates into external and internal flows, on the integral force characteristics is neglected.

It follows from experiments with solid axially symmetric bodies $(R \leqslant R_0)$ that the external boundary of a target of radius R_+ (restrained or open) has practically no effect if $R_+ > 5R_0$. On extrapolating this rough estimate to the case of the tube (ideally, by putting the effective boundary at $R_+ = \frac{1}{2}R_0$), we obtain that, when $h \leqslant h_0 < 0.1R_0$, the shapes of the caverns will only be slightly different on the outside and the inside. We emphasize that, in a solid, low porosity medium as well as a liquid, the cavern is formed by shearing actions. For instance, a fluid particle in the form of a cube on the axis ahead of the body around which the flow occurs assumes some peculiar shape after a certain time interval: its median line only approaches to the stagnation point, and points belonging to the upper and lower faces, by moving along the stream lines, bend around the figure and appear further downstream. The deformations around the axis are singular.

Optimization of a contour means that the body nose is rounded off so as to ensure, firstly, the maximum expansion of the free surfaces beyond the points of the flow separation, in the space between which an isolated element is inscribed in an optimal manner and, simultaneously, the minimum values of the hydrodynamic drag $C(M)$ (the minimum of the norm of the functions $C(M)$ in a certain class of functions in a mathematical sense). A compromise between these two opposing requirements is achieved, as it is in the case of rod-like configurations, depending on the actual case and possibilities available to the investigator. Here, it is obligatory to minimize the transverse forces and moments which arise without fail due to the asymmetry of the flow with respect to the median surface of the tube.

5. The law of constant plastic friction on the wetted surfaces of a projectile has traditionally been adopted in the treatment of experimental data [3, 7] and in theoretical papers [4, 5]. Generalizing this law, we introduce an effective mean (with respect to the space and time coordinate) frictional stress $\tau_0 \leqslant \tau$, as in the case when a thin viscous boundary layer is formed.

2. Starting out from these assumptions, we represent the resistance force acting on the body as viewed from the medium in the form of a sum of terms. One of these is the hydrodynamic drag. The others determine the dependence on the strength of the medium [1–4]

$$F = \tfrac{1}{2}C\rho_0 \upsilon^2 s + Bs + \tau^0 s_\perp; \quad s = \pi R_*^2, \quad 2\pi R_* h_* \tag{2.1}$$

where s and $s_\perp$ are the current values of the midsection of the wetted part of the body and the area of the projections of the wetted surface on the direction of the motion and B is a constant. Formula (2.1) has been confirmed in theoretical and experimental investigations [3–5]. No scale effect was observed in the case of clayey and sandy soils when $\upsilon_0 \sim (10\text{–}10^3)$ m/s. It is therefore assumed that the medium obeys the constitutive relationships of the von Mises model of elastoplasticity. The quasistatic resistance is then proportional to the constant yield stress [3–5] $B = b\tau$.

The right-hand side of (2.1) can be considered as the superposition of two asymptotic forms when $\upsilon_0 \to \infty$ (the strength forces are negligibly small) and when $\upsilon_0 \to 0$ (quasistatics). The strength forces disappear on passing to the limit of a liquid ($\tau \to 0$). When $M_0 = \upsilon_0 / c < 0.5$, the compressibility of the medium can be neglected and the quantity C may be assumed to be independent of the velocity. At transonic velocities, this dependence is already significant. Here, it is not fixed but the limits of the change in C are known: $0 < C < 2$.

The subsequent simplifications are associated with the replacement of the functions $C(M)$, $s(M)$, $s_\perp(M), \ldots$ by their mean values in the intervals $M > 1$ and $M_* < M < 1$, assuming that the variations in these quantities within the supersonic and hydrodynamic (subsonic) ranges of motion are small. Then, the integral of the ordinary differential equation for the motion of the body $F = mH^{\cdot\cdot}(t)$ (t is the time and m is the mass) with the initial conditions $H(0) = 0$, $H^{\cdot}(0) = \upsilon_0$ is taken in a finite form and the dependence of the finite depth of penetration on the velocity of entry reduces to the formula

$$H = \frac{H_0}{l} = \rho \sum_{i=1}^{3} \frac{k_i}{C_i} \ln \frac{1 + A_i M_{i-1}^2}{1 + A_i M_i^2} =$$

$$
\begin{aligned}
&= \varsigma_1 + \rho\frac{k_2}{C_2}\ln\frac{1+A_2}{1+n_*}\ (M_0>1),\quad H=\varsigma_2\quad (M_2<M_0<1)\\
&H=\frac{\rho}{C_3}\ln(1+A_3M_0^2)\ (M_0<M_2)\\
&\varsigma_i=\rho\frac{k_i}{C_i}\ln\frac{1+A_iM_0^2}{1+A_iM_i^2}+\frac{\rho}{C_3}\ln\left(1+n_*\frac{b_2C_3}{b_3C_2}\right)\\
&M_1=1,\ M_2=M_*,\ k_3=1,\ n_*=A_2M_*^2 \qquad (2.2)\\
&\rho=\frac{\rho_1}{\rho_0},\ A_i=\frac{C_i}{\delta b_i},\ b_i=b+\frac{s_{\perp i}\tau_i^0}{s_i\tau},\ \delta=\frac{2\tau}{E}\\
&\langle C\rangle=C_i,\ \langle h_*\rangle=h_i,\ \langle x_*\rangle=l_i,\ \langle s_\perp\rangle=s_{\perp i},\ \langle\tau^0\rangle=\tau_i^0\ (M_{i-1}<M<M_i)\\
&k_i=h_0/h_i,\ m=2\pi R_0h_0l\rho_1,\ s_i\approx 2\pi R_0h_i\ \text{(tube)}\\
&k_i=S/s_i,\ m=Sl\rho_1,\ s_i=\pi R^2(l_i)\ \text{(rod)}
\end{aligned}
$$

where R_0, h_0, l, $S=\pi R_0^2$ are the maximum radius, thickness of the tube, reduced length and midsection. In the derivation, use has been made of hypotheses 1–5 and the expressions mentioned above for the mass of the body m in terms of its density ρ_1. In order to estimate the quantity M_*, the number n_* is introduced. This number is equal to the ratio of the hydrodynamic drag to the corresponding strength forces at the arbitrary instant when the hydrodynamic regime changes to a state of flow without separation ($n_* \sim 10$). Formula (2.2) obviously remains valid when the interval (0, M_0) is divided up into an arbitrary number of segments. For a fixed mass and entry velocity, ρ, k_i, C and γ remain variable parameters. The asymptotic forms

$$H\sim\rho,\ k_i\quad \text{for}\quad k_i,\ \rho\to\infty \qquad (2.3)$$

which are indicative of the nature of the dependence on the basic parameters, follow immediately from (2.2).

The optimal sets of C_i are not strongly deformed and may point to another system of basic dimensionless quantities

$$\rho,\ k_i,\ n_0;\quad n_0=\rho_0 v_0^2/(2b_1\tau)\ (M_0>1)$$

where n_0 is the ratio of the initial hydrodynamic drag to the strength resistance of the medium.

In carrying calculations using formula (2.2), we select the values $k_1=k_2=k$ and ρ from the following physical considerations. The density of typical soils varies over a small range ($\rho_0=(1.5\text{–}3)\times10^3$ kg/m^3). Tungsten alloys (high dynamic strength, melting points, density and heat resistance) are the most suitable materials for the manufacture of penetrators which enter deeply into a terrestrial medium. Hence, $\rho=7.5$ can be taken when estimates are being made. No investigations whatsoever of the caverns at supersonic velocities (a liquid or solid medium) are known apart from a few experiments in sandy and clayey media (in the latter, the profiles of the residual caverns were photographed), where no special changes in cavern formation were noted on passing through the velocity of sound (private communication of Yu. K. Bivin). We shall therefore direct ourselves to well-known experiments and the results of calculations of stationary caverns in an incompressible fluid [8].

It is known that unsteady caverns (when braking occurs) are wider than steady ones. The same can also be said about caverns in a porous liquid. The compressibility, the initial stresses and strength introduce their own corrections and the problem of the calculation of caverns is therefore not simple at all. It is, however, clear that, if the parameter n_0 is sufficiently large, the laws governing the development of a cavern in a solid and in a liquid will be similar. Next, as the velocity decreases and strength forces play a greater role, the cavern in a solid medium will narrow down. In an incompressible liquid with a cavitation number equal to zero, the cavern expands to infinity and, theoretically, the parameter k may be as large as desired.

For the calculation let us take an elongation $L/2R_0 = 5$ (it will be around eight with respect to the mean diameter of the penetrator considered below). Then, in the case of a body which is inserted into a cavern behind a disc in a fluid, $R_0/R_* \approx 8$ [8]. For rough estimates of the depths of penetration into a solid medium we reduce this ratio by a factor of $(R_0/R_* = 4)$. The choice of this number is not completely arbitrary. The experiments noted above indirectly point to a value close to this. Moreover, the solution of a model problem on an expanding and moving cylinder in a densifying plastic medium [9] when matched with the asymptotic forms of the development of a cavern close to the edge of a disc, as in a liquid [8], does, in fact, yield such a lower estimate for the radius of the cavern $(n_0 \gg 1)$. Then, $k = 16$. Let us now turn our attention to the fact that $k = 1$ for a cylinder which moves along its own axis and this value is a minimum since $S \geqslant s$.

We will introduce the concept of an equivalent cylinder, the density (mass), reduced length and midsection of which are the same as those for the penetrator with a curvilinear generatrix. A physical meaning of the ratio of the resistive force acting on the cylinder which is equivalent to a penetrator with a disc-like nose shape to the resistive force acting on this penetrator under cavitating flow conditions when the body is inserted into a cavern without touching its domes can be assigned to the parameter k. It can be seen from the estimates that the magnitude of k may be quite large. This, and also the high value of the relative density ρ, according to the asymptotic form (2.3) result from a substantial increase in the depth of penetration (a decrease in the drag) compared with the analogous characteristics for bodies of "non-optimal shape". We shall subsequently refer to the parameter k as the optimality number of the shape of a body.

Planar caverns, however odd this is, differ only slightly according to the figure, from axially symmetric caverns [8, 10]. Correspondingly, for a tube, we take $k = 5$ for the reason that the flow remote from the site of separation expands approximately as in the case of a rod (in spite of the significantly greater elongation of the linear element) as a consequence of its stabilization at a certain distance from the separation point on account of the effect of the strength of the medium. The treatment of experiments on the penetration of cones with half aperture angles of 15°–90° into plasticine (the engineering model of a clayey medium) at velocities of 1–400 m/s $(\tau = \tau^0)$ showed that the magnitude of $B \pm s_\perp \tau / s$ is close to a constant 5×10^6 Pa at a value of $\tau = 2.2 \times 10^5$ Pa and that both of these quantities depend only slightly on the angle of the cone over a wide range of velocities [3, 7]. It follows from this that $b_{2,3} \approx 22$. We will use this value at the higher values of the velocities since, at high velocities $(M > M_*)$, on account of the flow separation, the replacement of τ^0 by τ is not significant and, under quasistatic conditions, $\tau = \tau_0$. Additionally, this estimate agrees with the theoretical result [5]. In the case of clays, sands and rocks, the magnitude of δ, with certain exceptions, oscillates from 10^{-3} to 5×10^{-3}.

The results of calculations using formulae (2.2) with the choice of the approximate mean values of the coefficients $C_1 = 1.2$, $C_2 = 0.8$ and $C_3 = 0.5$ (taking account of the tendency for these to increase as the Mach number increases) are shown in Fig. 1 (by the dashed lines for $n_* = 5$ and the solid lines for $n_* = 7.5$) for $\delta = 10^{-3}$ (curves 1), $\delta = 2.5 \times 10^{-3}$ (curves 2) and $\delta = 5 \times 10^{-3}$ (curves 3). Segments with small depths of penetration $(M_0 < M_*)$, where the majority of the experiments were carried out, and zones of rapid and then logarithmic growth of the depth with Mach number can be picked out in the graphs. The initial segments of slow growth, where the functions $H(M_0)$ are approximated by a linear dependence, correspond to quasistatic conditions. When the relative strength δ increases or there is a reduction in the coefficient C_2 (Fig. 2), they expand appreciably. The maximum depth H_* which is attained under quasistatic conditions is invariant with respect to the characteristics of the medium τ and E and is equal to ≈ 25 in the case of the parameters which were adopted (the solid lines in Fig. 1). The relative strength has an effect in a logarithmic manner. The main part of the graph $H = H(M_0)$ is displaced by $\approx K \times 50$ units downwards or upwards when δ is decreased or increased by a

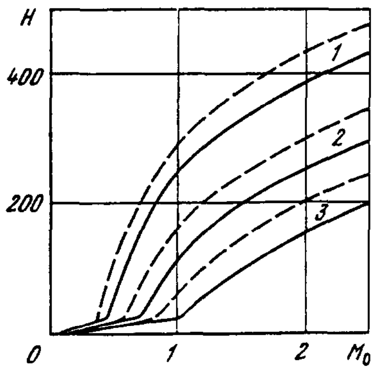

FIG. 1.

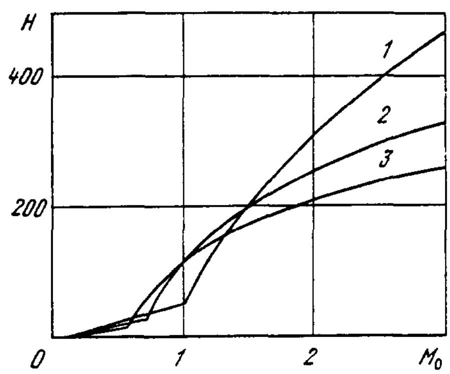

FIG. 2.

factor of K. An increase in the parameter n_* by a factor of one and a half led to a parallel transposition of the $H > H_*$ domain of the graphs downwards by ≈ 50 units. From physical considerations, a value $n_* = 7.5$ is more realistic (confirmation can be obtained during the treatment of the corresponding experiments). The results of varying the parameters C_i when $n_* = 7.5$ and $\delta = 2.5\times10^{-3}$ are shown in Fig. 2: when there is a uniform reduction in all of the previous values (curve 2) by two (curve 1) and they are increased by a factor of one and a half (curve 3). It is possible to pick out domains when there is a small variation of the function $H(M_0)$ close to the point $M_0 = 0$ where the hydrodynamic drag is not significant and, what is interesting, in the zone $1.2 < M_0 < 1.7$. This is associated with the variation in the parameter M_* when the coefficient C is changed and, correspondingly, with the prolongation of the quasistatic regime of the motion towards greater values of M_0 when C is decreased. Here, the curves intersect at the points $M_0 = 1$; 1.3. If only the coefficient C_1 is increased the curves coincide with curve 2 in the interval $0 < M_0 < 1$ and, when $M_0 > 1$, they lie between curves 2 and 3 $(1.2 \leqslant C_1 \leqslant 1.8)$ or slightly below curve 3 $(1.8 < C_1 \leqslant 2)$.

A knowledge of the relations $C = C(M)$ and the parameters of the problem in actual cases permits a more accurate prognosis of the result. The analysis carried out enables one to conclude that, when the shape and the material of the penetrator are optimized, relative depths $H \approx 120$ at $M_0 \approx 1$ and depths $H \approx 250$ at $M_0 \approx 2$ are actually attainable in soils of medium relative strength. The variation $10^{-3} < \delta < 5\times10^{-3}$ leads to the following scatter in the values $25 < H(M_0 = 1) < 250$ and $160 < H(M_0 = 2) < 390$.

3. With the indicated values of the numbers k and ρ, δ, $M_0 = \text{const}$, the dimensionless depths of penetration of a tube will be approximately one-third of the depth of penetration of a rod. A more exact relationship is given by formula (2.4), where the depth increment H_k is subtracted in the quasistatic regime since, in the case of a tube, this may be neglected

$$H_T \approx \tfrac{1}{3} H_c(1-\alpha), \quad \alpha = H_k / H_c \quad (H_k \approx 15 \div 25) \tag{2.4}$$

On account of the estimate (2.4), the results (Figs 1 and 2) are carried over to cases of the penetration of bodies of tubular shape. On the other hand, if one takes the ratio of the absolute depths, takes account of the approximate equality of the logarithmic terms and puts $k_1 = k_2$, $C_1 = C_2$ in formula (2.2), then one obtains

$$\frac{H_T}{H_c} \approx \kappa \frac{m_T}{m_c}, \quad \kappa = \frac{k_T}{k_c} \frac{R_0 / h_0}{2(1+\alpha)}$$

For the same masses and radii of a tube and a rod $H_T / H_c \approx \kappa$ and, even for relatively thick walled tubes $R_0 / h_0 = 1.5$, the gain in depth will be approximately double. Conversely, for the same depths of penetration, the ratio of the masses is equal to $m_c/m_T \approx \kappa$.

As an illustration, let us take a two-metre tube made of tungsten alloys ($m \approx 60$ kg, $h_0 = 5$ mm, $R_0 = 5$ cm). The relative depths of penetration of such a tube as a function of the strength of the soil and the initial velocity of entry are estimated by the quantities

$\tau\times10^{-5}$ Pa		10			30		100		500	
υ_0, km/s	0.5	1	1.5	1	1.5	2	2	3	2	3
H	22	72	102	32	61	83	38	68	0	10

and the maximum and normal stress, force and overload according to formula (2.1) are

$$\sigma_m \approx 10^9\upsilon_0^2 \text{ Pa}, \quad F_m \approx 1.6\times10^5\upsilon_0^2 H, \quad F_m/mg = 260\upsilon_0^2$$

(υ_0 is in km/s). The estimates were constructed under the assumption that $H \gg 1$; $\tau < 10$ corresponds to loose soils, moist clays and loams, $\tau < 30$ to sands, sandy loams, dry clays and loams, frozen soils and ice, $\tau < 100$ to weathered sandstones, limestones, tuffs, . . . , and $\tau < 500$ to rock salts, schists, tough limestones, sandstones, tuffs, ... It is more correct to assume that, in formulae (2.1) and (2.2), τ is a parameter of the process rather than of the medium on account of its inconstancy as a parameter of the medium and in view of the lack of a satisfactory description of the behaviour of a terrestrial medium over a wide range of loading rates and magnitudes of the deformations.

In the model which has been adopted, no account is taken of the dependence of the parameters on depth. Apart from heterogeneity, the geostatic stress–strain state has an effect which is sometimes extremely complex and not always with the compressive stresses in the horizontal plane. At the same time, the latter appreciably increases the drag on account of the strengthening of rocks [11]. Moreover, it follows from the experiments in [3, 7] that the dynamic strength τ exceeds the static strength by a factor of two to three in the case of clayey media. Obviously, the same coefficient should be taken into account in the case of highly broken down rocks and rocks which are not saturated with water. The coefficient will be smaller: 1–1.5 times in the case of water-saturated or strong rocks with a small number of defects. The prehistory, rate of loading and temperature turn out to have a considerable effect on the position of the yield surface. One of the components of the normal stresses is close to zero around an elongated penetrator. This is explained by the small strengthening of the medium during flow around a rigid body compared with experiments on multilateral compression. On the other hand, the phenomenon of the weakening of the medium in the pressure wave (for example, the thinning of soils and the breakdown of the structure when there is porosity) leads to a pronounced fall off in the magnitudes of τ and δ, and this means, to a substantial displacement of the graphs of the function $H(M_0)$ upwards. The hydrodynamic regime encompasses a wide range of velocities and the penetrator will preferentially move with minimum drag.

As a rule, the known natural experiments were carried out with penetrators of ogival shape. For instance, a projectile with a mass of 145 kg, a diameter of 0.152 m length of the equivalent cylinder (made of a tungsten alloy) $l = 0.42$ m travelling at an initial velocity of 706 m/s buried itself 76 m deep in the bottom of a dried-up lake. Consequently, the relative depth $H = 160$. Let us convert the depth to another mass, velocity and shape. Then, a projectile with a mass which is q times greater ($S = \text{const}$) penetrates into the same medium (with respect to its averaged characteristics) approximately q times deeper. A twofold increase in the velocity and refinement of the shape of the projectile, judging from the plots in Fig. 1, additionally approximately double the depth. This means that a projectile with a weight of about a ton with a velocity $\upsilon_0 = 1.4$ km/s travels a distance of about 2200 m in the above-mentioned medium. There is no doubt that inhomogeneity, initial stresses, and strength significantly reduce this value, but even then, it remains impressive. Experiments [13] have shown that such projectiles withstand the impact throughout a medium of sandstone type without fracture at velocities of 2.1 km/s, and even have reserves.

The formation of expanding caverns behind the body follows from the theory of flow around thin bodies by an elastic medium [14]. Cavitation traces, that is, channels with a cross-section $S_k > S$ have been observed a posteriori over the whole range of velocities investigated (up to $M_0 \sim 1$) with a significant increase in the mean cross-section of the channel as the level of the velocity increases and not only in moist clays [7] (Fig. 3) but also in sandstones, dry clays, frozen soils and in ice (private communication from

FIG. 3.

Yu. K. Bivin, who carried out these experiments). Naturally, channels do not remain in sands. However, simple technical solutions to the strengthening of the walls of the cavern during the motion of the penetrator are possible. The development of "terradynamic" technologies associated with deep penetration and the formation of a stable channel, that is, boreholes then becomes extremely promising. American investigators [15] have listed several of them.

4. Thus, on the basis of a number of physical hypotheses which can be theoretically substantiated and experimentally verified, formulae have been proposed for the integral characteristics of the penetration of rigid, elongated, blunt bodies into a plastic medium. The key idea, Lavrent'yev's hypothesis concerning the negligible smallness of solid effects in high velocity processes, is reflected in the description of the "zeroth approximation" to the shape of the body with minimum resistance. We emphasize that, generally speaking, the shape of the "optimal penetrator" is not needle-shaped as can be seen at first glance: the elongation of a conventional sewing needle is <50 while it is about ten in the case of the body described in Section 2. The results can serve as a basis when estimating the effectiveness of the penetration process and in choosing the optimal regime of the motion and configuration. The launching of massive penetrators using modern methods of acceleration can be carried out with different applied and scientific-investigative aims [15] right down to the study of the physicochemical properties of geological sections on other planets.

The estimates confirm intuitive reasoning concerning the energetically more favourable launching of hollow bodies compared with solid bodies. We would, however, emphasize the following fact. The optimality numbers k of the planar and axially symmetric contours are, respectively, proportional to the first power and the square of the ratio of the linear dimensions of the midsection of the body at the site of flow separation and, since the expansion of the caverns is almost identical in this and other cases, these numbers are substantially different. This means that the optimal planar body penetrates into the medium to a far smaller depth than an axisymmetric body of the same characteristic dimensions and density. Attempts at the optimization of planar bodies are therefore much less successful than attempts to optimize bodies of revolution and the calculated motion of a tube, in the approximation of a planar element when compared with the case of a rod, yields a more modest gain than would be expected. The conclusion to be drawn from this that spatial elongated forms, which can be subdivided into systems of planar elements and bodies of revolution will be less effective than axisymmetric contours. Another conclusion, concerning the advantage of spatial (star-shaped) forms of small elongation, was drawn, starting out from the asymptotic form of the solution as $M \to \infty$ in gas dynamics [16] and in a liquid [17]. Wave resistance then plays a significant role.

We emphasize that the treatment presented above refers to the case when $M \sim 1$. In a liquid, when $M \sim 1$, minimum resistance is obtained in the case of "stars" (and not in the case of the equivalent cones) only for sufficiently fine lobes with large angles at the vertex [17]. In a solid medium, friction introduces its own corrections. Moreover, elongated bodies possess a large inertia compared with short bodies and this means that spatial configurations are not promising for attaining maximum depths in a solid medium. The inhomogeneity of the stress state around the penetrator becomes an important factor under quasistatic conditions and the predominance of strength forces as well as in estimating the effectiveness of the breakdown of the obstruction (perforation). Spatial (polyhedral, polyclinic) forms, as has been repeatedly noted, possess the better potential possibilities in this sense and it is quite likely that the optimum lies somewhere between these configurations.

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